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Abstract — A method to measure elastic and dissipative properties of the constituents of a sandwich structure is proposed and validated. The method relies on the comparison between (a) the modal frequencies and dampings of a thick plate as predicted by an extended Rayleigh-Ritz procedure and (b) the their values as given by experimentation or numerical simulation. On real plates, a one-point measurement of free vibrations is sufficient, provided that a high-resolution modal analysis [1] is used [2]. For validation purposes, the experimental modal analysis is replaced by a finite-element model analysis (numerical measurement). Minimising the differences between the modal characteristics yields an estimation of the values of the elastic and dissipative material properties. Agreement between estimated and original mechanical parameters is shown to be good for the parameters which are influential in plate vibration.
Mots clés — Elasticity parameters estimation – Loss factor estimation – Thick-plate vibrations.

1 Introduction

Sandwich structures are often used in the industry because they can be made light and stiff. However, mechanical properties of sandwich plates may be difficult to predict accurately, particularly if damping is considered. Measurements on isolated samples of the constituents or of whole panels are often needed.

The problem of the estimation of solely the *elasticity* parameters of a homogeneous material using plate vibrations has been widely addressed using a *thick-plate* (see for example [3, 4, 5, 6]) and provides in-plane and out-of-plane information about the materials. The problem of estimating the *elasticity and damping* parameters by using point measurements [7, 8, 9, 10] has retained some attention, but in a *thin-plate* context only. In a *thick-plate* context, only methods involving *full-field* measurements are currently available [11, 12, 13]: they are very time-consuming or need sophisticated equipment.

In this paper, a previously presented method for estimating some complex moduli of elasticity of the constituents of sandwich structures [2, 14] is numerically validated. The proposed procedure yielding *in-plane* and *out-of-plane* elasticity and dissipation parameters of the constituents of the sandwich is schematically presented in Fig. 1. The analytical model of the sandwich panel is presented in Sec. 2. Based on this model, the numerical modal frequencies f_n^{Num} and dampings α_n^{Num} are derived by means of an extended Rayleigh-Ritz procedure (Sec. 3). In the validation procedure, the modal frequencies f_n^{FEM} and dampings α_n^{FEM} replace the experimental values; they are given by a finite-element model analysis of a known virtual plate (Sec. 4). Given the numerical and virtual experimental data, the optimisation procedure that estimates the elasticity and damping parameters of the constituents of the sandwich material is detailed in Sec. 5. The estimation results and a sensitivity analysis are shown in Sec. 6 and 7 respectively.

2 Mechanical model of sandwich panels

The sandwich panel consists in two identical skins and a core (Fig. 1). In the following, "panel" designates the physical structure whereas "plate" refers to the idealised structure made out of an equivalent homogeneous material. Parameters pertaining to the core, the skin, and the homogeneous material constitutive of the equivalent plate are respectively denoted with c, s, and H indexes. The thicknesses of the core, skins, and panel are h^{c}, h^{s}, and $h = h^{\mathrm{c}} + 2h^{\mathrm{s}}$ respectively.

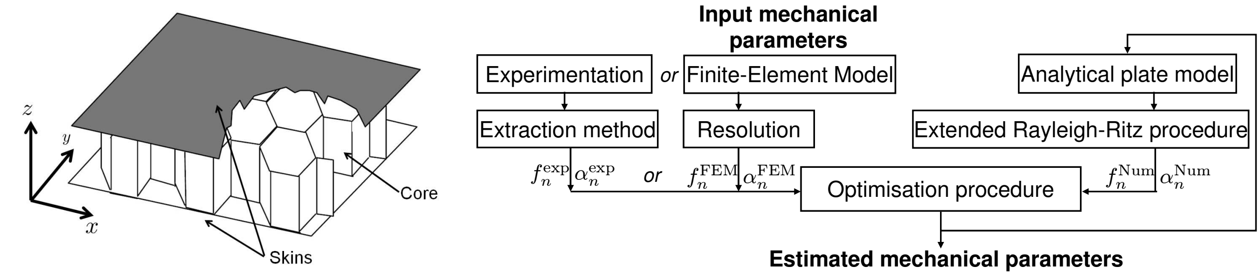

Figure 1: Left: geometry of the sandwich plate. Right: exeprimental and FEM-validation procedures.

The skin and core materials are considered as homogeneous, orthotropic in the x and y directions, and viscoelastic. The standard hysteretic model (which is frequency-independent) has been retained for describing the viscoelastic behaviour, with complex moduli $\underline{E} = E(1+j\eta)$. The stress-strain relationship $\boldsymbol{\sigma}^\gamma(\boldsymbol{\epsilon}^\gamma)$ in each $\gamma-$material ($\gamma =$s, c, or H) involves 7 complex numbers and can be written, to first order in η^γ as:

$$\boldsymbol{\sigma}^\gamma = \begin{bmatrix} E_x^\gamma(1+j\eta_x^\gamma) & \nu_{yx}^\gamma E_x^\gamma[1+j(\eta_{\nu_{yx}}^\gamma+\eta_x^\gamma)] & 0 & 0 & 0 \\ \nu_{xy}^\gamma E_y^\gamma[1+j(\eta_{\nu_{xy}}^\gamma+\eta_y^\gamma)] & E_y^\gamma(1+j\eta_y^\gamma) & 0 & 0 & 0 \\ 0 & 0 & G_{xz}^\gamma(1+j\eta_{xz}^\gamma) & 0 & 0 \\ 0 & 0 & 0 & G_{yz}^\gamma(1+j\eta_{yz}^\gamma) & 0 \\ 0 & 0 & 0 & 0 & G_{xy}^\gamma(1+j\eta_{xy}^\gamma) \end{bmatrix} \boldsymbol{\epsilon}^\gamma \tag{1}$$

The symmetry of the strain/stress relation implies $\nu_{xy}^\gamma E_y^\gamma = \nu_{yx}^\gamma E_x^\gamma$ and $\eta_\nu^\gamma = \eta_{\nu_{xy}}^\gamma + \eta_y^\gamma = \eta_{\nu_{yx}}^\gamma + \eta_x^\gamma$, leaving 12 independent real parameters to be identified for each material (24 altogether).

The following hypotheses are made on the sandwich panel:

– The sandwich panel is symmetric with respect to its mid-plane.
– Compared to the core, skins are thin enough to ensure that shear stress in the skin can be ignored: $h^{\mathrm{s}} G_{xz,yz}^{\mathrm{s}} \ll h^{\mathrm{c}} G_{xz,yz}^{\mathrm{c}}$.
– The core is much softer than the skins ($E_x^{\mathrm{c}} \ll E_x^{\mathrm{s}}$, $E_y^{\mathrm{c}} \ll E_y^{\mathrm{s}}$, $G_{xy}^{\mathrm{c}} \ll G_{xy}^{\mathrm{s}}$). Given the generic expression of the moduli of the homogeneous equivalent material $E^{\mathrm{H}} = \left(\frac{h^{\mathrm{c}}}{h}\right)^3 E^{\mathrm{c}} + \left[1 - \left(\frac{h^{\mathrm{c}}}{h}\right)^3\right] E^{\mathrm{s}}$, this ensures that all in-plane stress in the plate are entirely due to those in the skins.

According to these hypotheses, there is no stress associated with $\underline{E}_z^{\mathrm{c,s,H}}$, $\underline{\nu}_{xz}^{\mathrm{c,s,H}}$, $\underline{\nu}_{yz}^{\mathrm{c,s,H}}$, $\underline{G}_{xz}^{\mathrm{s}}$, $\underline{G}_{yz}^{\mathrm{s}}$, $\underline{E}_x^{\mathrm{c}}$, $\underline{E}_y^{\mathrm{c}}$, $\underline{G}_{xy}^{\mathrm{c}}$, $\underline{\nu}_{xy}^{\mathrm{c}}$, $\underline{\nu}_{yx}^{\mathrm{c}}$ which are ignored in what follows.

Under the hypothesis given in section 2, the sandwich panel behaves in the low frequency range like a homogeneous thick-plate [15]. The thickness of the plate is chosen to be h. Its mechanical properties are given in Eq. (2) as functions of the mechanical and geometrical properties of the skins and the core.

$$\begin{cases} E_x^{\mathrm{H}}(1+j\eta_x^{\mathrm{H}}) = E_x^{\mathrm{s}}(1+j\eta_x^{\mathrm{s}})\left[1-\left(\frac{h^{\mathrm{c}}}{h}\right)^3\right] & E_y^{\mathrm{H}}(1+j\eta_y^{\mathrm{H}}) = E_y^{\mathrm{s}}(1+j\eta_y^{\mathrm{s}})\left[1-\left(\frac{h^{\mathrm{c}}}{h}\right)^3\right] \\ \nu_{xy}^{\mathrm{H}}(1+j\eta_{\nu_{xy}}^{\mathrm{H}}) = \nu_{xy}^{\mathrm{s}}(1+j\eta_{\nu_{xy}}^{\mathrm{s}}) & G_{xy}^{\mathrm{H}}(1+j\eta_{xy}^{\mathrm{H}}) = G_{xy}^{\mathrm{s}}(1+j\eta_{xy}^{\mathrm{s}})\left[1-\left(\frac{h^{\mathrm{c}}}{h}\right)^3\right] \\ G_{xz}^{\mathrm{H}}(1+j\eta_{xz}^{\mathrm{H}}) = G_{xz}^{\mathrm{c}}(1+j\eta_{xz}^{\mathrm{c}}) & G_{yz}^{\mathrm{H}}(1+j\eta_{yz}^{\mathrm{H}}) = G_{yz}^{\mathrm{c}}(1+j\eta_{yz}^{\mathrm{c}}) \end{cases} \tag{2}$$

The 12 independent real parameters $\{E_x^{\mathrm{H}}, \eta_x^{\mathrm{H}}, E_y^{\mathrm{H}}, \eta_y^{\mathrm{H}}, G_{xy}^{\mathrm{H}}, \eta_{xy}^{\mathrm{H}}, G_{xz}^{\mathrm{H}}, \eta_{xz}^{\mathrm{H}}, G_{yz}^{\mathrm{H}}, \eta_{yz}^{\mathrm{H}}, \nu_{xy}^{\mathrm{H}}, \eta_\nu^{\mathrm{H}}\}$ are to be estimated. Their knowledge yields the elastic and dissipative properties of each layer of the sandwich panel provided that the system of 12 real equations formed by Eqs. (2) is invertible. Given the previous hypotheses, a sufficient condition is:

$$\eta_x^{\mathrm{c}} \frac{E_x^{\mathrm{c}}}{E_x^{\mathrm{s}}} \ll \eta_x^{\mathrm{s}} \quad \eta_y^{\mathrm{c}} \frac{E_y^{\mathrm{c}}}{E_y^{\mathrm{s}}} \ll \eta_y^{\mathrm{s}} \quad \eta_{xy}^{\mathrm{c}} \frac{G_{xy}^{\mathrm{c}}}{G_{xy}^{\mathrm{s}}} \ll \eta_{xy}^{\mathrm{s}} \tag{3}$$

This condition is not satisfied only if the η^{c}-coefficients are several orders of magnitude larger than the η^{s}-ones.

3 An extended Rayleigh-Ritz procedure for obtaining modal dampings

3.1 Modal representation

In the present study, it is assumed that structural damping is dominant over damping due to acoustical radiation and boundary conditions. The honeycomb sandwich panel is considered here as a system with N degrees of freedom $\boldsymbol{q} = \{q_m\}$, where $\boldsymbol{q}$ is any set of generalised displacements. The equation of its free motion is [16]:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = 0 \tag{4}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ are the mass, damping, and stiffness matrices. The link between $\boldsymbol{C}$ and the dissipative model presented previously will be made further. The associated conservative system (corresponding to $\boldsymbol{C} = 0$) has N normal modes with modal shapes $\boldsymbol{\xi}_n$ and normal frequencies f_n. If the complete, non-conservative, system is lightly damped, it can be shown [16] that, to first order, its so-called natural modes have the same modal shapes $\boldsymbol{\xi}_n$ and natural frequencies are $f_n + j\alpha_n$.

3.2 Potential, kinetic and dissipated energies in the equivalent thick-plate

Let U_n be the potential energy associated with the n^{th} natural mode of the dissipative plate for a maximum vibrational amplitude of 1 on the plate. It varies in time as $\exp(-2\alpha_n t)$. Since the conservative and non-conservative systems have the same modal shapes, the same potential energy is associated with their n^{th} mode, equal to the kinetic energy T_n of the n^{th} normal mode of the conservative system. The energy lost by this mode during one cycle ΔU_n is therefore:

$$\Delta U_n = -2\frac{\alpha_n}{f_n}U_n = -2\frac{\alpha_n}{f_n}T_n \tag{5}$$

Within the frame of the first order Reissner-Mindlin theory [17, Chap. 3], the displacements $\{u,v,w\}$ in the $\{x,y,z\}$-directions respectively can be written within a good approximation (see below) as:

$$u(x,y,z,t) = -z\Phi_x(x,y,t) \qquad v(x,y,z,t) = -z\Phi_y(x,y,t) \qquad w(x,y,z,t) = w_0(x,y,t) \tag{6}$$

The potential energy of the plate is:

$$\begin{aligned} U = \frac{1}{2}\iint_{\mathcal{S}} \Bigg[\quad & D_x\left(\frac{\partial \Phi_x}{\partial x}\right)^2 + D_{\nu_{xy}}\left(\frac{\partial \Phi_x}{\partial x}\frac{\partial \Phi_y}{\partial y}\right) + D_y\left(\frac{\partial \Phi_y}{\partial y}\right)^2 + D_{xz}\left(\Phi_x^2 - 2\Phi_x\frac{\partial w_0}{\partial x} + \left(\frac{\partial w_0}{\partial x}\right)^2\right) + \ldots \\ & D_{yz}\left(\Phi_y^2 - 2\Phi_y\frac{\partial w_0}{\partial y} + \left(\frac{\partial w_0}{\partial y}\right)^2\right) + D_{xy}\left(\left(\frac{\partial \Phi_x}{\partial y}\right)^2 + 2\frac{\partial \Phi_x}{\partial y}\frac{\partial \Phi_y}{\partial x} + \left(\frac{\partial \Phi_y}{\partial x}\right)^2\right)\Bigg]\mathrm{d}x\mathrm{d}y \end{aligned} \tag{7}$$

with

$$\begin{aligned} D_x &= \frac{E_x^{\mathrm{H}} h^3}{12(1-\nu_{xy}\nu_{yx})} \qquad D_{\nu_{xy}} = \frac{\nu_{xy}E_y^{\mathrm{H}} h^3}{6(1-\nu_{xy}\nu_{yx})} \qquad D_y = \frac{E_y^{\mathrm{H}} h^3}{12(1-\nu_{xy}\nu_{yx})} \\ D_{xz} &= 2\kappa_{xz}^2 h G_{xz}^{\mathrm{H}} \qquad D_{yz} = 2\kappa_{yz}^2 h G_{yz}^{\mathrm{H}} \qquad D_{xy} = \frac{G_{xy}^{\mathrm{H}} h^3}{6} \end{aligned} \tag{8}$$

The shear correction factors κ_{yz}^2 and κ_{xz}^2 account for the fact that Eq. (6) is an approximation: the (functional) angles Φ_x and Φ_y depend lightly on z and sections of the plate do not remain plane in the flexural deformation. The values $\kappa_{yz} = \kappa_{xz} = 1$ have been chosen according to the recommendations of [18] for sandwich panels.

By definition and based on the material model (Sec 2), the fraction of energy ΔU lost during one cycle $\mathcal{T}$ is:

$$
\begin{aligned}
\Delta U &= -\int_{\mathcal{T}} \iiint_{\mathcal{V}} (\boldsymbol{\sigma}^H)^T \frac{\partial \boldsymbol{\epsilon}^H}{\partial t} \, \mathrm{d}\tau \, \mathrm{d}t \\
&= -\pi \iint_{\mathcal{S}} \Bigg[\eta_x^{\mathrm{H}} D_x \left(\frac{\partial \Phi_x}{\partial x} \right)^2 + \eta_\nu^{\mathrm{H}} D_{\nu_{xy}} \left(\frac{\partial \Phi_x}{\partial x} \frac{\partial \Phi_y}{\partial y} \right) + \eta_y^{\mathrm{H}} D_y \left(\frac{\partial \Phi_y}{\partial y} \right)^2 + \\
&\quad \ldots \eta_{xz}^{\mathrm{H}} D_{xz} \left(\Phi_x^2 - 2\Phi_x \frac{\partial w_0}{\partial x} + \left(\frac{\partial w_0}{\partial x} \right)^2 \right) + \eta_{yz}^{\mathrm{H}} D_{yz} \left(\Phi_y^2 - 2\Phi_y \frac{\partial w_0}{\partial y} + \left(\frac{\partial w_0}{\partial y} \right)^2 \right) + \\
&\quad \ldots \eta_{xy}^{\mathrm{H}} D_{xy} \left(\left(\frac{\partial \Phi_x}{\partial y} \right)^2 + 2 \frac{\partial \Phi_x}{\partial y} \frac{\partial \Phi_y}{\partial x} + \left(\frac{\partial \Phi_y}{\partial x} \right)^2 \right) \Bigg] \mathrm{d}x\mathrm{d}y
\end{aligned} \tag{9}
$$

The kinetic energy T of the system is:

$$
T = \frac{\rho^{\mathrm{H}} \omega^2}{2} \iiint_{(\mathcal{V})} \left[u^2 + v^2 + w^2 \right] \mathrm{d}\tau = \frac{\rho^{\mathrm{H}} \omega^2}{2} \iint_{(\mathcal{S})} \left[\frac{h^3}{12} (\Phi_x^2 + \Phi_y^2) + h w_0^2 \right] \mathrm{d}x\mathrm{d}y \tag{10}
$$

where ρ^{H} is the density of the equivalent homogeneous thick plate: $h\rho^{\mathrm{H}} = h^{\mathrm{c}}\rho^{\mathrm{c}} + 2h^{\mathrm{s}}\rho^{\mathrm{s}}$.

3.3 Derivation of the modal dampings α_n^{Num}

In order to derive the modal dampings α_n from Eqs. 5, 9, and 10, analytical expressions of the Φ_x-, Φ_y-, and w_0-modal fields are needed. This is achieved by a Rayleigh-Ritz procedure for the normal modes of the associated conservative system since they are the same as those of the natural modes.

The generalised-displacement fields $\Phi_x(x,y)$, $\Phi_y(x,y)$, and $w_0(x,y)$ are projected on the elements of an orthonormal polynomial basis of order Q satisfying partially the free-free boundary conditions [19]:

$$
\Phi_x(x,y) = \sum_{i,j} L_{ij} p_i(x) p_j(y) \quad \Phi_y(x,y) = \sum_{i,j} M_{ij} p_i(x) p_j(y) \quad w_0(x,y) = \sum_{i,j} N_{ij} p_i(x) p_j(y) \tag{11}
$$

This procedure generates a new set of $N = 3Q^2$ generalised displacements L_{ij}, M_{ij} and N_{ij}. The next step consists in writing the kinetic and potential energies T and U. The Hamilton principle reads as:

$$
\forall (i,j) \in [0, Q-1]^2 : \; \frac{\partial (T-U)}{\partial L_{ij}} = 0 \quad \frac{\partial (T-U)}{\partial M_{ij}} = 0 \quad \frac{\partial (T-U)}{\partial N_{ij}} = 0 \tag{12}
$$

The above system of $3Q^2$ linear equations can be re-written as $[\boldsymbol{K} - 4\pi^2 f^2 \boldsymbol{M}]\boldsymbol{q} = 0$. The expressions of the partial derivatives of U with respect to L_{ij}, M_{ij}, N_{ij} yield $\boldsymbol{K}$ while the partial derivatives of T with respect to L_{ij}, M_{ij}, N_{ij} yield $\boldsymbol{M}$. The resolution of this eigenvalue problem gives a straightforward access to the modal frequencies f_n and modal shapes $\boldsymbol{\xi}_n$.

Introducing the modal coefficients $\boldsymbol{\xi}_n$ (expressed in the $\{L_{ij}, M_{ij}, N_{ij}\}$ system of coordinates) into Eqs. 11 yields the desired analytical expressions for the Φ_x, Φ_y, and w_0 modal fields and also for their x- and y-derivatives. For each of the N modes, the potential, lost, and kinetic energies can be written by introducing these expressions into Eqs. 7, 9, and 10:

$$
\forall n \in [1,N] : \quad T_n = 4\pi^2 f_n^2 t_n \quad U_n = \sum_{k=1}^{6} D_k u_k^n \quad \Delta U_n = -\pi \sum_{k=1}^{6} \eta_k D_k u_k^n \tag{13}
$$

where the subscripts $\{1,2,3,4,5,6\}$ stand for $\{x, \nu, y, xz, yz, xy\}$ respectively. The coefficients t_n and u_k^n depend on the geometry and mass parameters of the plate, and are quadratic in modal shapes $\boldsymbol{\xi}_n^{\mathrm{Num}}$.

The expression (14) of the modal dampings α_n^{Num} can be deduced from the Eq. (5) and the last two expressions of (13):

$$
\alpha_n^{\mathrm{Num}} = -\frac{f_n \Delta U_n^{NC}}{2T_n^C} = \frac{1}{8\pi f_n^{\mathrm{Num}} t_n} \sum_{k=1}^{6} \eta_k D_k u_k^n \tag{14}
$$

l_x (m)	l_y (m)	h (m)		ρ (kg/m^3)	
		Core	Skin	Core	Skin
0.4	0.6	4×10^{-3}	0.2×10^{-3}	40	700

Table 1: Geometry and constituent densities of the virtual sandwich-plate.

4 Modal frequencies and dampings of a virtual plate

Finite-element modelling and the associated computations have been performed using Cast3M [20], a free software developed by the French Centre for Atomic Energy (CEA). The model consists of 8-node quadratic 2D-thick-shell elements and assumes the Reissner-Mindlin hypothesis. Each node has 6 degrees of freedom. Elements are placed on a regular mesh of $N=60$ elements per side of the plates. The chosen sandwich plate is made of 3 homogeneous layers and is symmetrical with respect to its mid-plane. Geometrical, mechanical and mass parameters of the plate are given in Tab. 1 and in Fig. 2 (numbers).

Modal frequencies and dampings of the virtual plates are computed in two steps. At first, the modal frequencies of the conservative system f_n^{C} are computed by solving the eigenvalue problem $\boldsymbol{M}\ddot{\boldsymbol{q}}+\boldsymbol{K}\boldsymbol{q}=0$. In the second step, the non-conservative system is described according to the constitutive model of the material (section 2) which implies that $\boldsymbol{K}$ is a complex matrix $\boldsymbol{K}^*=\boldsymbol{K}+j\boldsymbol{K}'$. The dynamic equation of the dissipative system becomes $\boldsymbol{M}\ddot{\boldsymbol{q}}+\boldsymbol{C}\dot{\boldsymbol{q}}+\boldsymbol{K}\boldsymbol{q}=0$ with $\boldsymbol{C}=\boldsymbol{K}'/\omega$. The “light damping hypothesis” has been retained. The damping α_n is obtained as the real part of the n-th eigenvalue of this new problem where $\boldsymbol{C}$ is taken as $\boldsymbol{K}'/(2\pi f_n)$. Unlike in the usual modal analysis, the eigenvalue problem must be re-written (and solved) with a new parameter ($\boldsymbol{C}$) as many times as the number of complex eigenvalues to be found.

For the 3-layer virtual sandwich plate virtual plate, increasing the number of elements above 60 elements per side results in less than a 1 % relative variation of the 35 first modal frequencies (conservative and non conservative cases) and in less than 0.4 % of the 35 first modal dampings. Thus, 60 elements per side are enough to ensure the desired precision on the analysis of the first 35 modes of the two plates.

5 Optimisation procedure

This section describes how to derive, in two steps, the complex moduli of elasticity of the homogenised equivalent material of the sandwich plate $\{E_x^{\mathrm{H}},\eta_x^{\mathrm{H}},E_y^{\mathrm{H}},\eta_y^{\mathrm{H}},G_{xy}^{\mathrm{H}},\eta_{xy}^{\mathrm{H}},G_{xz}^{\mathrm{H}},\eta_{xz}^{\mathrm{H}},G_{yz}^{\mathrm{H}},\eta_{yz}^{\mathrm{H}},\nu_{xy}^{\mathrm{H}},\eta_{\nu}^{\mathrm{H}}\}$ from the virtual and numerical values of the modal frequencies and dampings f_n^{FEM}, f_n^{Num}, α_n^{FEM}, and α_n^{Num}.

The estimation of the elasticity parameters $\{E_x^{\mathrm{H}},E_y^{\mathrm{H}},G_{xy}^{\mathrm{H}},G_{xz}^{\mathrm{H}},G_{yz}^{\mathrm{H}},\nu_{xy}^{\mathrm{H}}\}$ is done by comparing the experimental and numerical modal frequencies. The estimation problem to solve is non-linear and several orders of magnitude are involved in the properties values. The following cost-function was used:

$$C_f=\sum_{n=1}^{N}\left(\frac{f_n^{\mathrm{FEM}}-f_n^{\mathrm{Num}}}{f_n^{\mathrm{FEM}}}\right)^2 \tag{15}$$

A simplex search method [21] (function *“fminsearch”* in Matlab$^{\mathrm{TM}}$) based on the rigidities $\{D_x,D_{\nu_{xy}},D_y,D_{xz},D_{yz},D_{xy}\}$ has been chosen. Estimation results obtained by this methods are known to be dependent on the initial values of the parameters. To minimise the influence of the starting point, the following initialisation strategy for the rigidities has been chosen:

1. The initial values of in-plane rigidities D_x, $D_{\nu_{xy}}$, D_y and D_{xy} are the most influential; they were derived from the three lowest modal frequencies of the panel, as proposed in [7].
2. The initial values of out-of-plane rigidities D_{xz} and D_{yz} are less critical; homogenisation theory proposed by Gibson [22] for honeycomb core sandwich panels is used. This theory requires a value for the elasticity moduli of the material composing the honeycomb core. The first estimation was based on static tests.

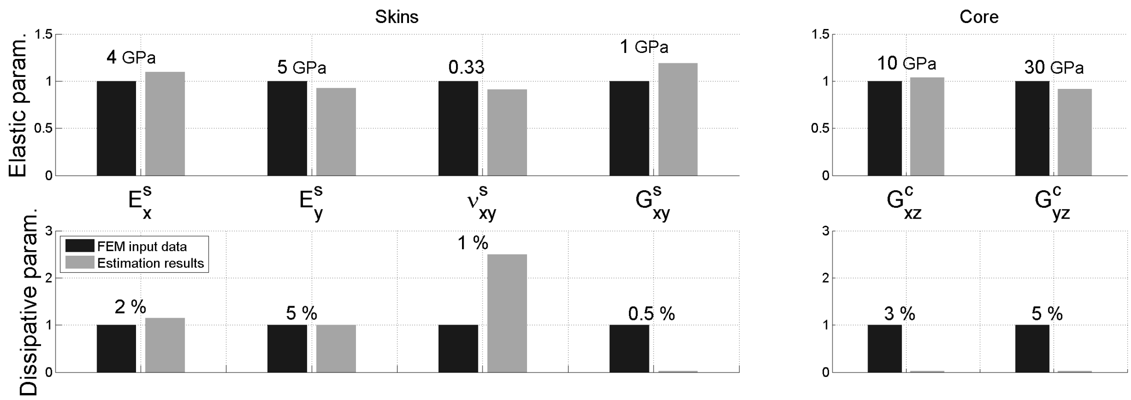

Figure 2: Comparison between the values of the mechanical parameters used in the FEM analysis ("FEM input data"), and their estimated values ("Estimation results") for each constituent (Skins, Core) of the virtual sandwich plate.

As can be seen in Eq. (14), modal dampings depend linearly on the loss factors $\{\eta_x^{\mathrm{H}}, \eta_y^{\mathrm{H}}, \eta_{xy}^{\mathrm{H}}, \eta_{xz}^{\mathrm{H}}, \eta_{yz}^{\mathrm{H}}, \eta_\nu^{\mathrm{H}}\}$ once the rigidities have been found. The estimation of the loss factors is therefore much easier than that of the elasticity parameters. A simple least-square optimisation procedure is sufficient to estimate the loss factors from the modal dampings. A cost-function similar to (15) has been chosen. The optimisation procedure is not iterative and needs no particular initialisation.

6 Estimation results for the 3-layer virtual sandwich-plate

The 3-layer virtual sandwich-plate has been used to validate the estimation procedure described in Fig. 1. Based on the first 35 modal frequencies given by the FEM and using a Rayleigh-Ritz order $q = 16$, the estimated values of the elasticity parameters are compared to the original values given to the FEM. The loss factors have been estimated with 28 modes and a model order $q = 18$. The estimated mechanical parameters are presented in Fig. 2 for each layer of the sandwich.

The residual mismatch between the results of estimation and the original values is discussed here. The mean absolute value $\left\langle \left| \frac{\Delta f_n}{f_n} \right| \right\rangle$ of the relative difference between experimental and numerical modal frequencies is 2.6 %. For the dampings, the residual mismatch $\left\langle \left| \frac{\Delta \alpha_n}{\alpha_n} \right| \right\rangle$ is 21.6 % in average but widely different between coefficients. These orders of magnitude suggest that the assumption that a 3-layer sandwich plate can be modelled as a simple homogeneous thick plate is correct in the frequency range under study.

It can be seen in Fig. 2 that the agreement between estimated and original parameters is globally very good. In-plane elasticity parameters of the skins and out-of-plane elasticity parameters of the core are estimated with a mean absolute relative error of 10.2 %. Principal in-plane loss-factors η_x and η_y are estimated with a comparable accuracy of 7.5 %. The imaginary part of $\underline{\nu}_{xy}^H$ is largely overestimated while the imaginary part of $\underline{G}_{xy}^H$ is underestimated. However, the overestimation of one parameter may be the result of the underestimation of the other, by compensation. The imaginary parts of $\underline{G}_{xz}^H$ and $\underline{G}_{yz}^H$ are assigned zero values by the estimation process. This underestimation is due to the fact that only a marginal part of the total energy-loss per cycle is dissipated through the mechanical couplings described by $\underline{G}_{xz}^H$ and $\underline{G}_{yz}^H$. As a consequence, modal dampings factors are not very sensitive to the loss factors described by their imaginary parts.

7 Sensitivity analysis

The sensitivities of the modal frequencies f_n or dampingsα_n to the coefficients $\{E_x^{\mathrm{H}}, \nu_{xy}^{\mathrm{H}} E_y^{\mathrm{H}}, G_{xz}^{\mathrm{H}}, G_{yz}^{\mathrm{H}}, G_{xy}^{\mathrm{H}}\}$ (for the modal frequencies) and to the corresponding loss factors $\eta_{\dots}^{\mathrm{H}}$ (for the modal dampings)

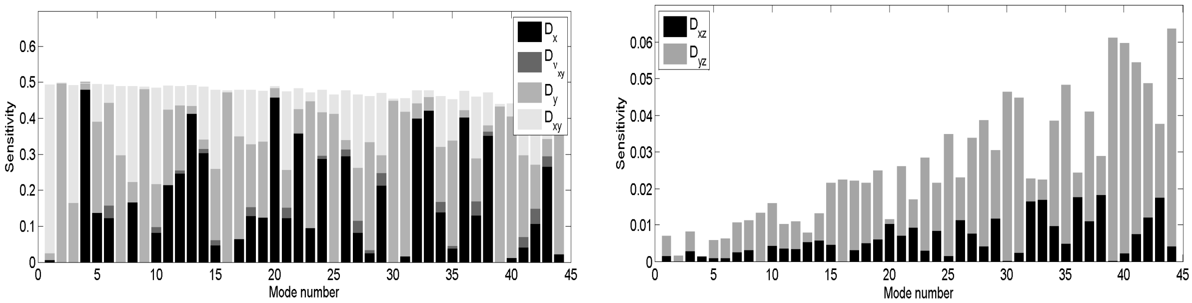

Figure 3: Sensitivities of the modal frequencies to the real parts of the rigidities given by Eqs. 8. Each column is the sum of the sensitivities to D_x, $D_{\nu_{xy}}$, D_y, and D_{xy} (left frame) and to D_{xz} and D_{yz} (right frame). The total may exceed 0.5 since some sensitivities are negative. the different involved parameters.

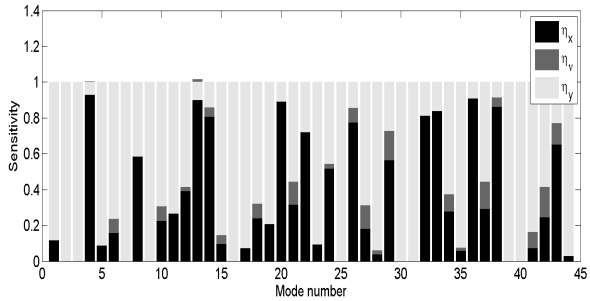

Figure 4: Sensitivities of the modal dampings to the dissipative parameters of the fictious material of the equivalent homogenised plate η_x^{H}, $\eta_{\nu_{xy}}^{\mathrm{H}}$, η_y^{H}, and η_{xy}^{H}.

are defined as $S_{\beta_n}(X) = \dfrac{\partial \beta_n}{\partial X}\left(\dfrac{\beta_n}{X}\right)^{-1}$: if the X parameter is increased by 1%, the n-th modal value β_n is increased by $S_{\beta_n}(X)$%. The sensitivities reflect the information contained in a modal value relatively to a given material parameter.

Results on modal frequencies with regard to the rigidities D_x, $D_{\nu_{xy}}$, D_y, D_{xy} (which depend on the elastic parameters as given by Eqs. 8) are presented in the left frame of Fig. 3 whereas the sensitivities to the out-of-plane rigidities D_{xz} and D_{yz} are presented in the right frame. Since the modal frequencies are very little sensitive to the Poisson coefficients, the corresponding sensitivities have not been represented. As expected, it turns out that modes of the form $(0,i)$ or $(j,0)$ convey a lot of information relatively to E_x and E_y respectively (left part of Fig. 3). Since the thick-plate model differs from the thin-plate model in the high-frequencies only, it is normal that there is almost 10 times more information relative to G_{xz} and to G_{yz} in the higher modes than in the lower ones (right part of Fig. 3). The lower sensitivity of the modal frequencies to G_{xz} than to G_{yz} is simply due to the aspect ratio of the plate ($l_x < l_y$).

Results on the sensitivities of modal dampings to the dissipative parameters of the fictious material of the equivalent homogenised plate (η_x^{H}, $\eta_{\nu_{xy}}^{\mathrm{H}}$, η_y^{H}, and η_{xy}^{H}) are given in Fig. 4. Modal dampings are sensitive to all in-plane parameters whereas out-of-plane parameters η_{xz}^{H} and η_{yz}^{H} were found to be irrelevant.

8 Conclusion

An easy-to-implement method to measure all relevant elastic and dissipative properties of the constituents of a sandwich structure has been described. Its experimetnal implementation has been presented elsewhere [2]. For the sake of numerical validation, a finite-element model replaces the experimental modal analysis. The sensitivity analysis shows that, within the homogenisation hypotheses, only half of all elastic and dissipative parameters are relevant for the vibration of thick plates. The method appears as reliable for their determination. The method does not give access to the other parameters but on the other hand, they are of little practical importance as far as the vibration of sandwich plates is concerned. Compared to the method by De Visscher *et al.* [8], it gives access to much higher frequencies and thus, to out-of-plane parameters. It is much simpler to implement and faster to perform than the method by Pagnacco *et al.* [11] and to that of Matter *et al.* [12], based on measurements of the vibration on the whole panel.

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